

# Scalable Bayesian Deep Learning for Uncertainty Quantification



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**THE THROUGHLINE**

One idea, **factor the weights low-rank**, makes Bayesian deep nets **efficient and geometrically singular**; balance the factors and the posterior **certifies its own centre**.

$W = AB^T$  cuts cost  $O(mn) \rightarrow O(r(m+n))$  and concentrates the posterior on a measure-zero manifold; balanced SVD factors then certify the deterministic centre; and for any Bayesian model, MI decomposes into per-class  $C_k$ .

THREE PAPERS · ONE PROGRAM

## 01 THE GEOMETRY & EFFICIENCY

### Singular BNNs

The posterior concentrates on a rank- $r$  manifold, a singular object, not just a cheaper one. Tighter rank-aware bounds; improved OOD detection across benchmarks.

## 02 THE GUARANTEE

### Certifying the Center

Balanced SVD factors remove gauge ambiguity, so the learned posterior gives a representation-invariant PAC-Bayes certificate for its deterministic center.

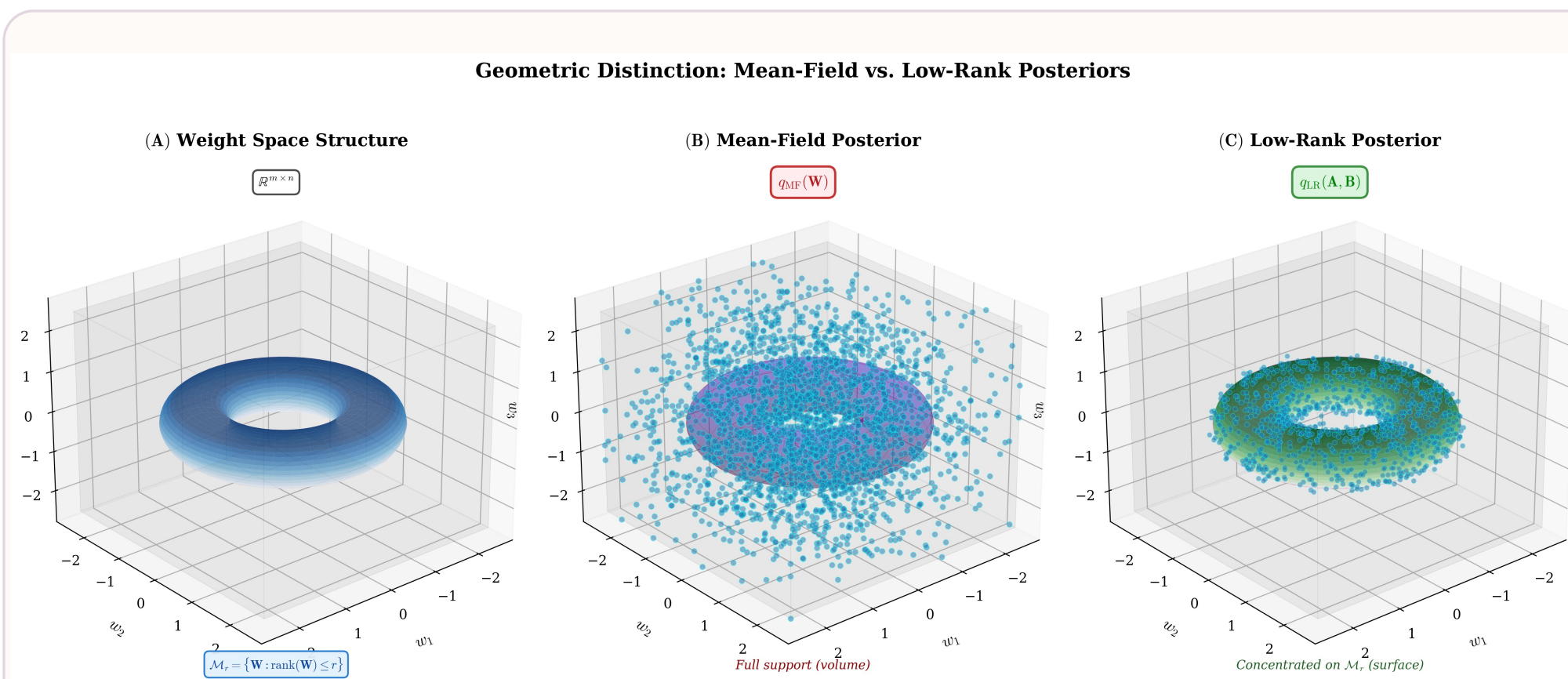
## 03 THE MEANING

### Where, Not Just How Much

Split scalar MI into per-class contributions  $C_k$ , exposing which class the model is unsure about, cutting selective risk on critical tasks.

PAPER 01 · SINGULAR BAYESIAN NEURAL NETWORKS

## The posterior support has changed



Mean-field fills the volume. Low-rank lives on the surface, a measure-zero rank- $r$  manifold.

**CENTRAL THEOREM**

If  $r < \min(m,n)$  and  $W = AB^T$ :

$$q_W(\mathcal{R}_r) = 1 \quad \lambda(\mathcal{R}_r) = 0 \Rightarrow q_W \perp \lambda$$

The posterior has no density over weight space.

**0.802**

**AUC-OOD**  
best of all methods

**33×**

**fewer params**  
vs Deep Ensemble

Across MLP · LSTM · Transformer and three distribution-shift regimes

Task	Arch.	Params	In-dist.	OOD	Headline
MIMIC-III mortality	MLP	13.6 K	AUROC 0.895	0.802	best OOD separation
Beijing PM <sub>2.5</sub>	LSTM	47 K	MAE 10.63	0.710	best coverage (PICP 0.790)
SST-2 sentiment	Transf.	1.47 M	Acc 0.806	0.640	≈ Deep Ens. at 8× less time
<b>Low-Rank SBNN (ours)</b>	<b>all 3</b>	↓	<b>matches FR</b>	<b>↑ vs FR</b>	<b>quality-efficiency win</b>

PAPER 02 · CERTIFYING THE DETERMINISTIC CENTER

## From a learned posterior to a guarantee

**STEP 1**

### The three objects

One low-rank model induces three certifiable predictors: the Gibbs posterior, the predictive mean, and a deterministic center. Each needs a different theorem.

**STEP 2**

### The obstruction

Factor gauge:  $(A,B)$  and  $(cA, c^{-1}B)$  give the same  $W$  but different norms, a naive factor-space certificate is representation-dependent.

**STEP 3**

### The repair

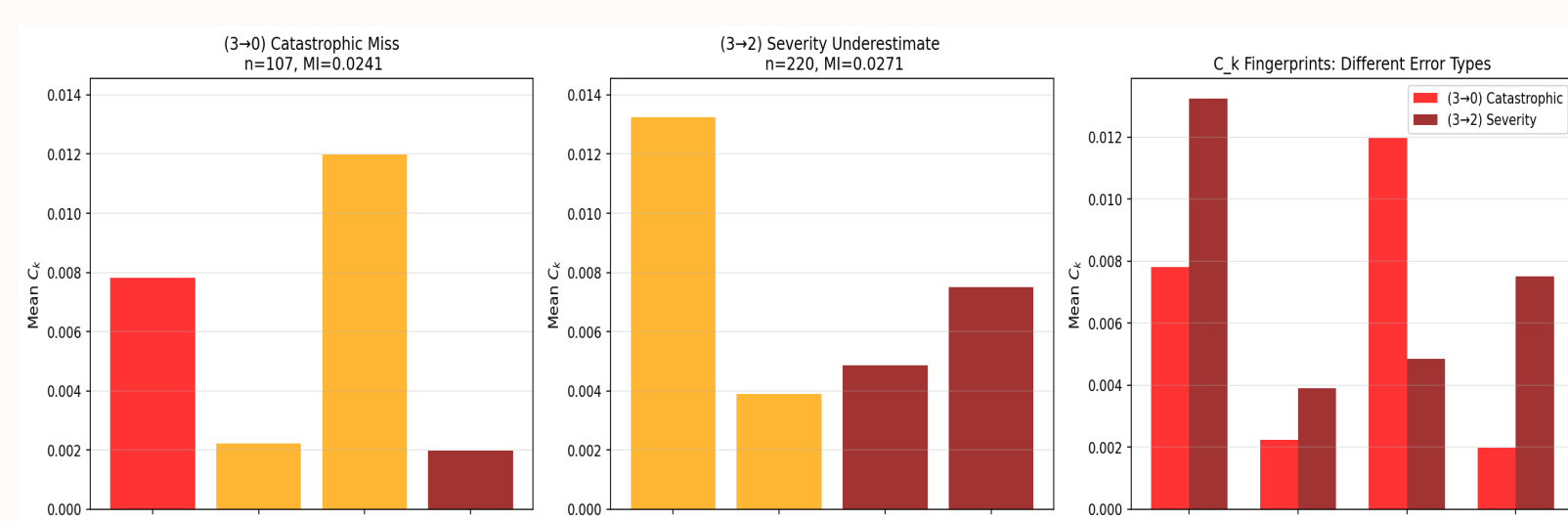
Pass to balanced factors from the SVD of the center weights. The certificate becomes intrinsic to the realized weights, gauge-free.

**RESULT**

A **representation-invariant PAC-Bayes margin certificate** for the deterministic center: explicit conditions on the learned scales  $\sigma_A, \sigma_B$  make the posterior noise Neyshabur-admissible, the posterior itself becomes the certifying perturbation law.

PAPER 03 · DECOMPOSING EPISTEMIC UNCERTAINTY

## Where the model is unsure, not just how much



Ck fingerprints differ by error type — catastrophic misses (3→0) load C<sub>2</sub>; severity errors (3→2) load C<sub>0</sub>. Same MI, different structure.

**34.7%**

lower selective risk vs MI on diabetic-retinopathy deferral

**56.2%**

lower selective risk vs variance baselines

**best**

AUROC for OOD;  $\sum_k C_k$  exposes asymmetric shifts MI hides

**THE SYNTHESIS**

**Three results compose:** low-rank factorization induces geometric singularity and efficiency; balanced SVD factors make the posterior certify its deterministic center; and per-class  $C_k$  decomposes any Bayesian model's uncertainty by class, exposing where the model is ignorant.